THE DIFFERENCE METHOD OF SOLVING THE CONJUGATE PROBLEM OF HEAT EXCHANGE DURING GAS FLOW IN A THICK-WALLED CHANNEL BETWEEN COMMUNICATING VESSELS

G. T. Aldoshin, V. I. Zhuk,
B. É. Kért, and K. M. Shlyakhtina

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An algorithm is developed for the numerical solution of a nonlinear system of differential equations describing the processes of gasdynamics and heat exchange during the transfer of gas from vessel to vessel through a flat, annular, or cylindrical channel with walls of finite thickness.

Recently a number of problems of modern technology — the regulation of heat exchangers, the creation of systems for the supply of energy to a hot gas, systems for the "refrigeration" of pipelines and vessels, etc. — have required the creation of methods for calculating conjugate problems of heat exchange: The joint solution of the equations of convective heat transfer in the fluid and the equation of heat transfer in the channel wall of the channel (vessel). The joining of the solutions at the fluid—wall boundary in an exact formulation must be carried out using boundary conditions of the fourth kind. In the case of turbulent flow of a compressible gas the possibility of obtaining solutions of this kind is problematical because of difficulties arising in the determination of the temperature field in the fluid. But since in engineering applications one usually needs knowledge of the stream parameters averaged over the cross section and of the temperature distribution in the channel walls, one can use one-dimensional equations of hydraulics to describe the flow and make the solutions conjugate using boundary conditions of the third kind [1]. The problem was analyzed in such a formulation in [2] for a channel with thermally thin walls and in [3] for a channel with walls of finite thickness and with assigned conditions at the channel entrance.

The problem of determining the temperature field in the walls of the channel and of the vessels and of determining the stream parameters during the transfer of gas from vessel to vessel or during the escape of gas from a vessel through a long channel is analyzed in the present report. A diagram of the problem is presented in Fig. 1. The vessels Ω_1 and Ω_2 are connected by a flat, annular, or cylindrical channel Ω_3. The annular and the flat axisymmetric channels are bounded by the walls Ω_4 and Ω_5; in the case of cylindrical and of flat symmetrical channels, the inner wall Ω_5 is absent. The walls of the vessels consist of a finite number of sections $\Omega_1{}^k$, $\Omega_2{}^k$, $k = 1, 2, \ldots$, each of which can be flat ($\sigma k = 0$), cylindrical ($\sigma k = 1$), or spherical ($\sigma k = 2$). The distribution of the parameters in the entire region of calculation is known at the initial time. Starting from this time a gas, whose flow rate and heat content are known functions of time, enters the volume Ω_1. The subsequent variation of the temperature field in the walls of the structure and of the parameters of the gas is calculated under the following assumptions: The processes of filling and emptying of the vessels are assumed to be quasistatic; heat conduction in the direction normal to the surface is allowed for in the walls of the vessels; heat conduction in the normal and axial directions is allowed for in the walls of the channels; the thermophysical properties of the materials of the walls of the channel and the vessels are known functions of the temperature and the coordinates; boundary conditions of the first, second, or third kind are set up at the outer surfaces Γ_i, $i = 3, 6, 7, 8$, of the walls of the channel and the vessels (henceforth, for determinacy, boundary conditions of the first kind are considered); the gas is assumed to be ideal; the heat capacity, thermal conductivity, and molecular weight of the gas depend on the temperature.

The processes in the vessels are described by the following equations: For the first law of thermodynamics (1), of mass balance in a vessel (2), of state (3), and the heat-conduction equation for sections of the walls of the vessels (4):

$$\frac{d}{dt} m_i U_i = -P_i \frac{dV_i}{dt} + G_i I_i + (-1)^i F_3 \rho_{3i} u_i I_{3i} - F_{wi} q_{wi}^k + Q_i; \tag{1}$$

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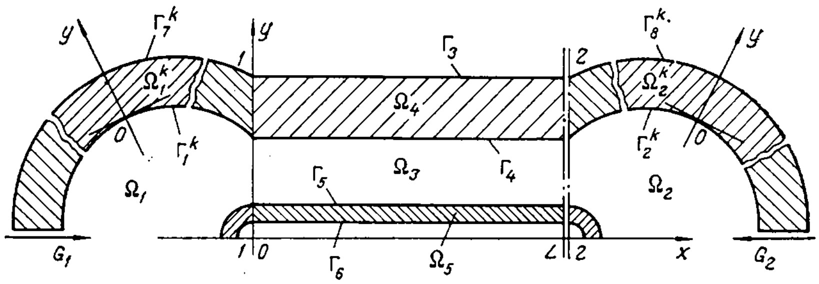

Fig. 1. Calculating diagram.

$$\frac{dm_i}{dt} = G_i + (-1)^i F_3 \rho_{3i} u_i; \tag{2}$$

$$P_i V_i = m_i R(T_i) T_i; \tag{3}$$

$$\rho_i^k (y, T) c_i^k (y, T) \frac{\partial T_i^k}{\partial t} = -y^{-\sigma k} \frac{\partial}{\partial y} \left(y^{\sigma k} \lambda^k (y, T) \frac{\partial T_i^k}{\partial y} \right). \tag{4}$$

The conditions of conjugation with respect to the temperature in the form of boundary conditions of the third kind (5) are set up at the inner surfaces $\Gamma_1{}^k$ and $\Gamma_2{}^k$ of sections of the walls of the vessels. The boundary conditions (6) are assigned at the outer surfaces $\Gamma_7{}^k$ and $\Gamma_8{}^k$:

$$-\lambda_i^k \frac{\partial T_i^k}{\partial y} = \alpha_i^k (T_i - T_i^k (0, t)) \equiv q_{wi}^k; \tag{5}$$

$$T_i^k (H^k, t) = T_a^k (t). \tag{6}$$

With known parameters of flow at the inlet and outlet of the channel the system of equations (1)-(6) is closed by the choice of criterial functions for the heat-exchange coefficients and by the assignment of the time dependences of the volumes of the vessels and the areas of sections of the walls, as well as the dependence of the flow rate G_2 at the exit from the sink vessel Ω_2 on the parameters of the gas in and outside the vessel and on time:

$$\mathrm{Nu}_i^k = \varphi_i^k (\mathrm{Gr}, \mathrm{Pr}, \ldots); \quad V_i = V_i (t); \quad F_{wi}^k = F_{wi}^k (t); \\ G_2 = G_2 (T_2, P_2, P_{a2}, t, \ldots). \tag{7}$$

In the system of equations (1)-(7) i = 1, 2; ρ_{3i}, u_i, and I_{3i} are the density, velocity, and heat content of the gas at the inlet (i = 1) and exit (i = 2) of the channel; the flow rate is positive during the filling and negative during the emptying of the vessels. The initial conditions are arbitrary:

$$m_i (0) = m_{i0}; \quad T_i (0) = T_{i0}; \quad T_i^k (y, 0) = T_{i0}^k (y). \tag{8}$$

The one-dimensional nonsteady flow of a compressible fluid in a channel of constant cross section, with allowance for the radial and axial heating of the channel wall, is described by the system of equations

$$\frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} + P \frac{\partial u}{\partial x} - \frac{P}{RT} \left(R + T \frac{dR}{dT} \right) \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) = 0; \tag{9}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{RT}{P} \frac{\partial P}{\partial x} - \xi \frac{u^2}{2d_э}; \tag{10}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{\varkappa - 1}{\varkappa} \cdot \frac{T}{P} \left(\frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} + \frac{4}{d_э} (q_5 + q_6) \right) + \xi \frac{u^2}{2d_э c_p}; \tag{11}$$

$$\rho_i (X) C_i (X) \frac{\partial T_i}{\partial t} = y^{-\sigma} \frac{\partial}{\partial y} \left(y^\sigma \lambda_i (X, T) \frac{\partial T_i}{\partial y} \right) + \frac{\partial}{\partial x} \left(\lambda_i (X, T) \frac{\partial T_i}{\partial x} \right), \tag{12}$$

where $\sigma = 0$ for a flat channel and $\sigma = 1$ for an axisymmetric channel. The conjugation conditions (13) are set up at the surfaces Γ_4 and Γ_5 of contact between the gas and the channel walls, while the boundary conditions (14) are set up at the outer surfaces Γ_3 and Γ_6 of the channel walls:

$$q_i \equiv (-1)^{i-1} \lambda_i (T) \frac{\partial T_i}{\partial y} = \alpha_i (x,\ t)(T_i (x,\ 0,\ t) - T^* (x,\ t)), \tag{13}$$

where

$$i = 5,\ 6;\quad T^* = T + u^2/2c_p;\quad T_i (x,\ H_i,\ t) = T_{ai} (x,\ t),\ i = 5,\ 6. \tag{14}$$

The connection between the flow parameters at the inlet and outlet of the channel and the parameters of the gas in the vessels is described in detail in [2]. At the inlet to the channel the laws of conservation of momentum (15) and energy (16) are used for this purpose. At the outlet from it the law of conservation of momentum (17) is used for the subcritical mode of discharge, while for the critical mode the velocity is equated with the local velocity of sound (18):

$$P_1 = \left(P + \frac{P}{RT} u^2\right)_{x=0}; \tag{15}$$

$$\frac{2\varkappa_1}{\varkappa_1 - 1} R_1 T_1 = \left(\frac{2\varkappa}{\varkappa - 1} RT + u^2\right)_{x=0}; \tag{16}$$

$$P_2 = \left(P + \delta \frac{F_3}{F_2} \frac{P}{RT} u^2\right)_{x=L} \quad \text{for} \quad u|_{x=L} < A|_{x=L}; \tag{17}$$

$$u|_{x=L} = \sqrt{\varkappa RT}|_{x=L} \quad \text{for} \quad u|_{x=L} = A|_{x=L}. \tag{18}$$

The boundary conditions for Eqs. (12) at the ends 1—1 and 2—2 of the channel walls are set up as follows. In the case of smooth conjugation of the channel wall with the adjacent section of the vessel wall the temperature of the end is assumed to equal the temperature of this section. If the channel conjugates with the vessel at a right angle, then the boundary conditions at the sections of the boundaries Γ_3 and Γ_6 of the channel walls which are in contact with the vessel walls are set up in the same way, while boundary conditions of the third kind in the form (13) are set up at the ends. If the channel is sufficiently long, then heat conduction through the ends can be neglected and they can be considered as thermally insulated.

With known parameters in the vessels the system of equations (9)-(18) is closed by the choice of the criterial functions (19) for the coefficients of convective heat exchange and hydraulic resistance:

$$\mathrm{Nu}_i = \mathrm{Nu}_i (\mathrm{Re},\ \mathrm{Pr},\ \ldots);\quad \xi = \xi(\mathrm{Re},\ \ldots). \tag{19}$$

The effect of the nonsteadiness of the flow and heat exchange on the coefficient of heat transfer is allowed for in accordance with the recommendations of [4]. In the system (9)-(11) the index i = 3, corresponding to the number of the region from which the parameter is taken, is omitted, as is done later for the parameters of the flow in the channel; in Eqs. (12)-(19) i = 5, 6; $\delta = 0$ for discharge from the channel into the atmosphere $\delta = 1$ for discharge into the sink vessel; the initial conditions are arbitrary:

$$T_i (X,\ 0) = T_{i0} (X);\ u (X,\ 0) = u_0 (X);\ P (X,\ 0) = P_0 (X);\ T (X,\ 0) = T_0 (X). \tag{20}$$

The nonlinear system of differential equations (1)-(20) is approximated by a difference system on a grid, which is constructed as follows. The grid is nonuniform in time and includes both whole and fractional layers. The grid is nonuniform with respect to the transverse coordinate y in the walls of the channel and the vessels and is bunched toward the surfaces of contact between the gas and the walls. The grid is uniform along the longitudinal coordinate x in the channel. The derivative of the product on the left side of (1) is expanded, the derivative of the internal energy is expressed through the derivative of the temperature, the pressure is eliminated from the right side of (1) using (3), and the Bernoulli equation is used to express the heat content. After substitution of (2) into (1) the system (1), (2) is written in vector form:

$$\frac{d}{dt} \overline{X}_i = \overline{f}_i, \tag{21}$$

where $\overline{X_i} = (T_i,\ m_i)$; $\overline{f_i} = (f_{i1},\ f_{i2})$; f_{i1} is the right side of the transformed equation (1); f_{i2} is the right side of (2). The system (21) is approximated by the Runge—Kutta system of the second order of accuracy [5]:

$$2(\bar{X}_{i(N)}^{m+\frac{j}{2}} - \bar{X}_{i(NN)}^{m+\frac{j-1}{2}})/\Delta t_{m+1} = (\bar{f}_{i(N-1)}^{m+\frac{j}{2}} + \bar{f}_{i(NN)}^{m+\frac{j-1}{2}})/2, \quad j = 1,\ 2. \tag{22}$$

The heat-conduction equation (4) is approximated by the nonlinear implicity difference system (23) with a time step of 0.5 Δt_{m+1}. The conjugation condition (5) is approximated on the grid by replacing the derivative by the difference equation (24):

$$y_s^{\sigma k}(\rho_i^k c_{i(N-1)}^{k,m+\frac{j}{2}})_h T_{i\bar{t}}^{k,m+\frac{j}{2}} - (y_s^{\sigma k}\lambda_{ih(N-1)}^{k,m+\frac{j}{2}} T_{iy(N)}^{k,m+\frac{j}{2}})_{\bar{y}} = 0,$$

$$X \in \Omega_{1h}^k \cup \Omega_{2h}^k,\ k = 1,\ 2,\ \ldots; \tag{23}$$

$$-\lambda_{ih(N-1)}^k T_{iy(N)}^{k,m+\frac{j}{2}} = \alpha_{i(N)}^k (T_{i(N)}^{m+\frac{j}{2}} - T_{i(N)}^{k,m+\frac{j}{2}}),\quad X \in \Gamma_{1h}^k \cup \Gamma_{2h}^k. \tag{24}$$

In (21)-(24) i = 1, 2. Here and later the index (N) denotes the number of the iteration of the parameters in which the value of the grid function is calculated. The index (NN) in (22) equals the number of the last iteration of the parameters in the vessels in the preceding whole layer for j = 1, while NN = N for j = 2. The system of hydraulic equations is approximated by a nonlinear implicit difference system of the first order of accuracy, which is written linearly relative to the iteration of the parameters being calculated and coincides with that in [2] in the calculation of the first iteration:

$$u_{\bar{t}(N)}^{m+1} + u_{(N-1)}^{m+1} u_{\bar{x}(N)}^{m+1} - \left(\frac{RT}{P}\right)_{(N-1)}^{m+1} P_{\bar{x}(N)}^{m+1} + \left(\frac{\xi u}{2d_e}\right)_{(N-1)}^{m+1} u_{(N)}^{m+1} = 0; \tag{25}$$

$$P_{\bar{t}(N)}^{m+1} + u_{(N-1)}^{m+1} P_{\bar{x}(N)}^{m+1} + P_{(N-1)}^{m+1} u_{\bar{x}(N)}^{m+1} -$$

$$-\left\{\frac{P}{RT}\left(R + T\frac{dR}{dT}\right)\right\}_{(N-1)}^{m+1} (T_{\bar{t}(N)}^{m+1} + u_{(N-1)}^{m+1} T_{\bar{x}(N)}^{m+1}) = 0; \tag{26}$$

$$T_{\bar{t}(N)}^{m+1} + u_{(N-1)}^{m+1} T_{\bar{x}(N)}^{m+1} - \left(\frac{\varkappa - 1}{\varkappa}\frac{T}{P}\right)_{(N-1)}^{m+1} (P_{\bar{t}(N)}^{m+1} + u_{(N-1)}^{m+1} P_{\bar{x}(N)}^{m+1})$$

$$-\left\{\frac{\varkappa - 1}{\varkappa}\frac{T}{P}\frac{4}{d_e}(q_5 + q_6)\right\}_{(N-1)}^{m+1} - \left(\frac{\xi}{2d_e c_p} u^2\right)_{(N-1)}^{m+1} u_{(N)}^{m+1} = 0. \tag{27}$$

The heat-conduction equation (12) is approximated by an economical, nonlinear, uniform difference system of fractional steps (i = 5, 6)

$$\frac{1}{2} y_s^{\sigma} (\rho_i c_{i(N-1)}^{m+\frac{1}{2}})_h T_{i\bar{t}(N)}^{m+\frac{1}{2}} - (y_s^{\sigma} \lambda_{i(N-1)h}^{m+\frac{1}{2}} \beta T_{ix(N)}^{m+\frac{1}{2}})_{\bar{x}} = 0; \tag{28}$$

$$\frac{1}{2} y_s^{\sigma} (\rho_i c_{i(N-1)}^{m+1})_h T_{i\bar{t}(N)}^{m+1} - (y_s^{\sigma} \lambda_{i(N-1)h}^{m+1} \beta T_{iy(N)}^{m+1})_{\bar{y}} = 0. \tag{29}$$

The same notation as in (1)-(20) is retained in the system of difference equations (23)-(29) for the grid analogs of the unknown functions. The index h denotes the transfer of the coefficients of the initial heat-conduction equations to the grid. In the case when the coefficients of Eqs. (4) and (12) are continuous their grid analogs are obtained by simple transfer to the grid. In the case of discontinuous coefficients the grid analogs must be obtained with the help of Steklov averagings [6] over the cells of the grid. In this case the statement of the problem (1)-(20) must be supplemented by the conditions of conjugation in the form of boundary conditions of the fourth kind at the discontinuity lines. The index m is equal to the number of the time layer, m = 0, 1, ..., M − 1. The index s is equal to the number of the grid node along the transverse coordinate y. The following notation is used for the grid function W:

$$\beta W^{m+\frac{j}{2}} = \frac{W^{m+\frac{j}{2}} + W^{m+\frac{j-1}{2}}}{2};\quad W_{\bar{t}}^{m+1} = \frac{W^{m+1} - W^m}{\Delta t_{m+1}};$$

$$W_{\bar{t}}^{m+\frac{j}{2}} = \frac{W^{m+\frac{j}{2}} - W^{m+\frac{j-1}{2}}}{\Delta t_{m+1}/2}; \tag{30}$$

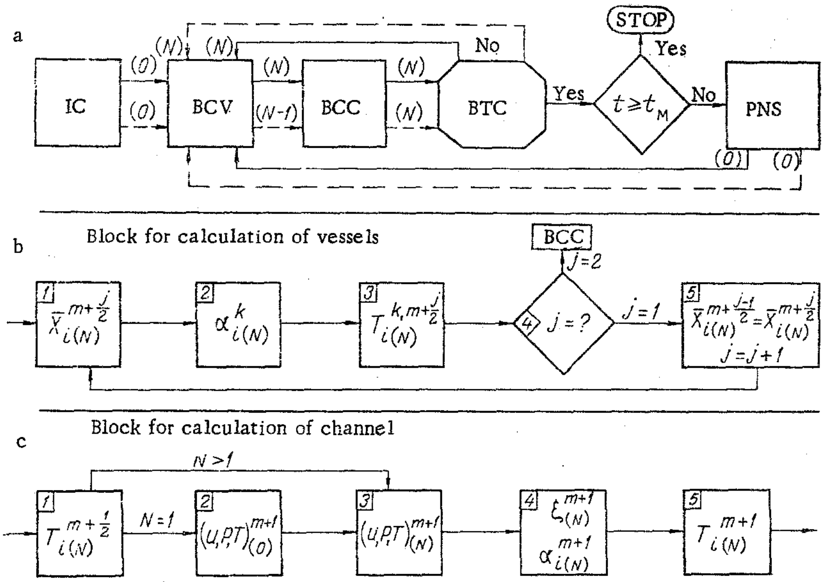

Fig. 2. Block diagram of the method: IC: initial conditions; BCV: block for calculation of vessels; BCC: block for calculation of channel; BTC: block for testing of convergence; PNS: preparation of next step; dashed arrows: parameters of channel; solid arrows: parameters of vessels.

$$W_{\bar{x}} = \frac{W_r - W_{r-1}}{H1}; \quad W_{\tilde{y}} = 2\,\frac{W_s - W_{s-1}}{H2_s + H2_{s-1}};$$

$$W_y = \frac{W_{s+1} - W_s}{H2_{s+1}}; \quad W_x = \frac{W_{r+1} - W_r}{H1}.$$

The grid approximation of the conjugation conditions (13) is constructed in the same way as (24). The grid analogs of the initial and boundary conditions (6), (8), (14), and (20) are obtained by their simple transfer to the grid if they are continuous. If the functions assigning the initial and boundary conditions are discontinuous then Steklov averagings are used to transfer them to the grid. The conditions (15)-(18) of conjugation of the parameters of the gas in the channel and the vessels are approximated by simple transfer to the grid and are written linearly relative to the (N)-th approximation of the unknowns in the channel. This makes it possible, as in [2], to use them as the boundary conditions in the solution of the system (25)-(27) by the orthogonal trial-run method [7]:

$$\left(\frac{m_1}{V_1} R_1 T_1\right)^{m+1}_{(N)} = \left\{P^{m+1}_{(N-1)} + \left(\frac{P}{RT} u\right)^{m+1}_{(N-1)} u^{m+1}_{(N)}\right\}_{x=0}; \tag{31}$$

$$\left(\frac{2\varkappa_1}{\varkappa_1 - 1} R_1 T_1\right)^{m+1}_{(N)} = \left\{\left(\frac{2\varkappa}{\varkappa - 1} R\right)^{m+1}_{(N-1)} T^{m+1}_{(N)} = u^{m+1}_{(N-1)} u^{m+1}_{(N)}\right\}_{x=0}; \tag{32}$$

$$\left(\frac{m_2}{V_2} R_2 T_2\right)^{m+1}_{(N)} = \left\{P^{m+1}_{(N)} + \delta \frac{F_3}{F_2} \left(\frac{P}{RT} u\right)^{m+1}_{(N-1)} u^{m+1}_{(N)}\right\}_{x=L}$$

$$\text{for} \quad u^{m+1}_{(N)}\big|_{x=L} < \left\{(\varkappa RT)^{m+1}_{(N)}\right\}^{\frac{1}{2}}_{x=L}; \tag{33}$$

$$u^{m+1}_{(N)}\big|_{x=L} = \left\{\left(\frac{\varkappa R}{u}\right)^{m+1}_{(N-1)} T^{m+1}_{(N)}\right\}_{x=L} \quad \text{for} \quad u^{m+1}_{(N)}\big|_{x=L} \geqslant \left\{\varkappa RT)^{m+1}_{(N)}\right\}^{\frac{1}{2}}_{x=L}. \tag{34}$$

An iteration algorithm was developed for the solution of the nonlinear system of algebraic equations (22)-(34). All the equations of the system are written linearly relative to iteration (N) of the unknowns. In the process the coefficients of the equations are calculated in the (N − 1)-th iteration of the unknowns. The first iteration of the parameters in the first step in time is taken from the initial conditions, while in succeeding steps it is taken from the preceding time layer. The process of calculation of iteration (N) when iteration (N − 1) is known is shown in Fig. 2a. The formulation of the (N − 1)-th approximation of the parameters in the vessels and the channel in Eqs. (22)-(24) allows one to calculate the (N)-th approximation of the parameters of the vessels. The use of the (N)-th approximation of the parameters of the vessels allows one to calculate the (N)-th approximation of the parameters in the channel. The process is repeated until the criterion of convergence of the iterations is satisfied, after which the transition is made to the calculation of the next step in time, etc., up to the selection of the time interval. The calculation of the (N)-th approximation of the parameters of the vessels in each cycle of iterations i carried out in the following sequence (Fig. 2b): 1) One step is made by the Runge—Kutta system (22) and the (N)-th approximation of the parameters of the gas in a vessel is calculated in a fractional layer; 2) the (N)-th approximation of the parameters of the gas and the (N − 1)-th approximation of the temperature of the vessel walls are substituted into the criterial functions (7) and the (N)-th approximation of the coefficients of heat transfer in the vessels is calculated; 3) the calculated values of $\alpha^{k}_{i(N)}$ and $T^{m+1/2}_{i(N)}$ are substituted into the conjugation conditions (24), after which they are used as the boundary conditions in the solution of Eqs. (23) by the trial-run method, which allows one to calculate the (N)-th approximation of the temperature $T^{k,m+1/2}_{i(N)}$ in the sections of the vessel walls; 4) the operations 1), 2), and 3) are repeated for the entire time layer, after which the transition is made to the calculation of the parameters in the channel in each cycle of iterations is this (Fig. 2c): 1) Eqs. (28) are solved by a trial run over the longitudinal coordinate, during which one uses as the boundary conditions either the values of $T^{k,m+1/2}_{i(N)}$ calculated earlier, or the analogs of Eqs. (24) for the ends of the channel, or the ends are assumed to be thermally insulated; 2) for N = 1 the zeroth approximation of the flow parameters $(u, P, T)^{m+1}_{(0)}$ is assigned from the conditions in the preceding whole time layer, while for N > 1 the (N − 1)-th iteration of the flow parameters is known; 3) the system of equations (25)-(27) with the boundary conditions (31)-(34) is solved by the orthogonal trial-run method and the (N)-th approximation of the flow parameters and the (N − 1)-th approximation of the temperature of the channel walls in the whole layer are substituted into the criterial functions (19) and the (N)-th approximation of the coefficients of heat transfer $\alpha^{m+1}_{i(N)}$, and hydraulic resistance $\xi^{m+1}_{(N)}$ is calculated; 5) the calculated values of the coefficients of heat exchange and the flow parameters in the channel are substituted into the difference analog of the conjugation conditions (13), after which they are used as the boundary conditions in the solution of Eqs. (29) by a trial run over the transverse coordinate and the calculation of $T^{m+1}_{i(N)}$. With this the calculation of the channel ends and the criteria of convergence of the iterations are tested.

An advantage of the iteration algorithm described is that it is easily generalized to the case of a system of vessels connected by channels.

The algorithm presented was realized in the form of an ALGOL program for a BÉSM-6 computer. The results of trial calculations permit the conclusion that the proposed method is efficient and is applicable to the calculation of processes in technological devices whose scheme is reduced to that discussed here.

NOTATION

m, mass of gas in a vessel; U, internal energy; P, pressure; V, volume; G, gas flow rate; I, total enthalpy; T, temperature; u, velocity; ρ, density; λ, thermal conductivity; F, useful cross-sectional area of gas stream; q, heat flux; Q, amount of heat released in a vessel per unit time; F_W, area of heat-exchange surface; R, gas constant; c, specific heat capacity of wall material; t, time; x, longitudinal coordinate; y, transverse coordinate; H, wall thickness; α, coefficient of heat transfer; ξ, coefficient of hydraulic resistance; d_e, hydraulic diameter; $\varkappa$, adiabatic index; c_p, specific heat capacity of the gas with p = const; A, velocity of sound; H1, step along the longitudinal coordinate; H2, step along the transverse coordinate; Δt, step in time; X, point of the plane; Nu, Nusselt number; Pr, Prandtl number; Gr, Grashof number; Re, Reynolds number. Indices: i, number of region; k, number of section of vessel wall; a, surrounding medium; 0, initial; N, number of iteration; m, number of step in time; r, number of grid node along the longitudinal coordinate.

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METHOD OF DESCRIPTIVE REGULARIZATION AND QUALITY OF APPROXIMATE SOLUTIONS

V. A. Morozov, N. L. Gol'dman, and M. K. Samarin

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A method of solving Fredholm integral equations of the first kind is described, which is based on the a priori knowledge of the arrangement of extrema and inflection points of the desired solution and permits taking account of the fundamental qualitative regularities inherent in the exact solution of the problem.

1°. The mathematical theory of the solution of incorrectly formulated problems has been developed sufficiently well at this time [1, 2]. The main point of this theory is the use of a priori information about the accuracy of giving the entrance data and (or) about the desired solution to some extent. The nature of this information can be twofold: quantitative or qualitative. As a rule, the majority of methods use quantitative information about the accuracy of giving the entrance data and quite general information about the "smoothness" of the solution (the Tikhonov regularization method, the residual method). The distinctive peculiarity of the Ivanov method of quasisolutions is the possibility of using not only information of the type mentioned, but also just qualitative information associated with the a priori representations of the behavior of the desired solution. As a rule, an objective basis for the presence of such information is intuitive considerations about the simplicity of the structure of the desired solution as well as certain general conceptions about the behavior of the physical process being studied. The former are related to the natural tendency of the researcher to identify the most important and essential items in the mathematical model and can also be dictated by fully defined esthetic considerations.

The latter appear, for example, when a perfectly evident fact in the study of the brightness distribution of a star is the drop in intensity from the center of the star to its edges if, certainly, the star is unitary, and the presence of two maxima if the star is binary.

Let us assume that the phenomenon being studied is characterized quantitatively by the function $u = u(x)$, $a \le x \le b$. Such quantitative characteristics as the variation in the function $u(x)$, the root-mean-square value of its k-th derivative, etc., which are often used in solving incorrect problems, can be taken as a measure of its "simplicity." It is also well known that the behavior of a function is modeled sufficiently effectively on an intuitive level if the possible arrangement of its characteristic points, extremum points, and the change in curvature is given. It is hence considered that, on the whole, the function will behave in a natural manner, i.e., is single-valued, has no reentrant points, is sufficiently smooth, and therefore, can be drawn with one "stroke" of the pen. Such a class of simple functions can be given if sections of their monotonicity and convexity are indicated. The class of smooth functions with L − 1 sections of monotonicity can be written by the conditions

$$M = \{u(x) : (-1)^{l+i} u'(x) \leqslant 0,\ x_i \leqslant x \leqslant x_{i+1},\ i = 1, 2, \ldots, L-1\},$$

where x_i, $i = 1, 2, \ldots, L$ are extrema of the function $u(x)$, $a = x_1 < x_2 < \ldots < x_L = b$ and the parameter l, equal to 1 or 2, governs the nature of the monotonicity in the first section. It is hence assumed that $M = M(x_2, \ldots, x_{L-1}; l, L)$, i.e., the number of extrema, the alternation of sections of growth and decrease in the function, and also the arrangement of the inner extrema can vary. Great detail in the class of functions being considered will be achieved if sections with curvature of constant sign are also extracted. We then arrive at the class

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